

# The iPhone Goes Downstream: Mandatory Universal Distribution\*

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## Abstract

Apple's decision to market iPhones using a single downstream vendor has prompted calls for Mandatory Universal Distribution (MUD), whereby Apple would have to sell to all potential vendors. We show that an upstream monopoly might want to use one or more downstream vendors, and society might benefit or be harmed by MUD. However, if the income elasticity of demand for the new good is greater than the income elasticity of the existing generic good, we find that MUD leads to a higher equilibrium price for both the new good and the generic, and therefore lowers consumer welfare.

*Keywords:* vertical restrictions, mandatory universal distribution, new product

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# 1 Introduction

Apple allows only one U.S. wireless phone provider, AT&T, to distribute its iPhone. Consumer organizations such as Consumers Union want the government to require that the iPhone be available through many or all downstream providers. In 2009, a Senate antitrust panel held hearings and Senators listed steps that they wanted the FCC and the Department of Justice to take to make the downstream industry more competitive (Consumer Reports 2009). The issue of mandating universal distribution (MUD) has arisen in many markets (e.g., movies and other durables) in the past and will arise with new, disruptive inventions. One of the chief arguments by proponents of MUD appeals to the conventional wisdom that equilibrium price falls when there are more firms in the market. It is, of course, well-known that if the upstream industry is competitive, then in many settings, raising the number of oligopolistic downstream firms lowers the price to final consumers. However, this relation may not hold if the upstream provider is a monopoly that can adjust its wholesale price to downstream firms depending on the number of vendors that carry its product. We present conditions under which MUD hurts or helps consumers and society.

We assume that there is a vertical industry structure, with two types of upstream firms. A monopoly produces a new product (e.g., Apple's iPhone). A competitive industry produces a generic product. Downstream, a quantity-setting oligopoly sells the generic and some or all of the firms also sell the new product.

We use a two-stage model. In the first stage, an upstream monopoly that has a new product decides whether it wants one or more downstream vendors to distribute its product. It engages in a game with the selected downstream firm or firms to determine whether they will carry the product. The upstream firm charges a constant wholesale price, but charges a downstream firm a lump-sum fee for the right to carry its product.

In the second stage, the downstream firms use a fixed-proportion production process: they resell the phones to final consumers. Both downstream firms sell a generic product. One or more of the downstream firms also sells the new product. The firms choose how many units of the new and the generic product they sell and a Cournot equilibrium results.

We focus on three issues that determine the outcome. First is the degree of substitution in consumption between the new and the generic products. Second is the size of the fixed cost of enabling each vendor to sell the new product. (For example, AT&T incurred substantial cost in enabling the iPhone to work on its network.) Third is the bargaining game that determines the number of downstream vendors and the wholesale price.

For a specific bargaining game, we show that each of four outcomes is possible for some combination of fixed cost and degree of substitution. The upstream firm may want to sell to only

one downstream firm and consumers may be better off with either one or two (or more) vendors downstream. Similarly, the upstream firm may want to sell to two (or more) downstream firms and consumers may or may not be better off with one vendor.

We start by examining two special cases, where imposing MUD is undesirable. In the first special case, the new and the generic product are not substitutes and downstream firms are oligopolistic. In the second special case, the downstream firms behave as price takers and the goods may or may not be substitutes.

We then turn to our main model in which the goods are substitutes and the downstream firms are oligopolistic. For simplicity, we assume that there are two downstream firms. We start with a particular first-stage bargaining game and investigate the equilibrium effect of the degree of substitution and the size of the fixed cost. We then discuss alternative bargaining games.

Our model differs from most models of market power in vertical markets by having an upstream monopoly sell a product that competes with another product downstream, allowing the upstream firm to charge both a per unit price and to extract a lump-sum transfer, examining contracting between the upstream and downstream firms, and providing an analysis of the MUD policy. While there are many papers that examine vertical relations, some that consider substitution between products downstream, and a few that examine bargaining issues, we are unaware of any paper that cover all facets of our paper and no other paper that examines the relevant policy question.

Our results have the flavor of several literatures that show upstream firms can use vertical contracts (or vertical integration) to soften competition by raising rivals' costs (possibly through sabotage) or foreclosing entry strategically (Salop and Scheffman 1983, Aghion and Bolton 1987, Economides 1998, Ordober, Salop and Saloner 1990, Hart and Tirole 1990, Riordan 1998, Weisman 2001, White 2007, and Bustos and Galetovic 2008). In many of these articles, vertical foreclosure occurs because an upstream firm controls some "essential facility" or "bottleneck resource" that competing firms need access to at comparable prices to compete downstream. A key point in many of these papers is that firms can collectively earn only a single monopoly profit so that the upstream firm, by charging a monopoly price for access to the essential facility, can extract all of the monopoly rents without further harming consumers. Our paper captures this same insight, though we focus on a model in which the upstream monopoly captures rents through a transfer payment as well as through per-unit charges. Some of the foreclosure literature examines government policies that forbid explicit foreclosure, which is similar to a MUD policy. However, our results differ from most of that literature because our upstream product competes with another product downstream.

Although our models differ, some of our results resemble those of the strategic trade literature (Spencer and Brander 1983), where the upstream firm's actions allow a single downstream

vendor to commit and gain a strategic advantage over its rival. One other literature that is similar in spirit to our paper concerns wholesale non-discrimination rules. MUD could be viewed as a particular non-discrimination rule that forbids setting the price to some downstream firms prohibitively high. Indeed, in an argument analogous to the current debate, Bork (1978) suggested that total welfare would increase if new markets are served due to wholesale price discrimination. However, these papers look at a very different vertical model. Typically in these models, the upstream monopoly wants to discriminate because the downstream firms have different costs or serve markets with different demand elasticities (Schmalensee 1981, Varian 1985, Katz 1987, De-Graba 1990, Ireland 1992, Yoshida 2000, and Villas-Boas 2009). We abstract from those considerations by assuming that all downstream firms are identical and all sell in the same market. This simplification allows us to focus attention on the strategic reasons for selling to one or more downstream firms, reasons not discussed in detail in the discrimination literature.

## 2 Special cases

In two special cases, MUD does not affect consumer welfare but lowers industry profit and hence social welfare. In the first special case, the upstream monopoly sells a good that is not a substitute for existing goods. We assume that the downstream firms incur no additional cost of selling the good. There are a limited number of potential downstream firms,  $N$ . When more than one firm sells the new good, the outcome is a Cournot equilibrium.

If the upstream monopoly's only instrument is its wholesale price then it has the traditional double marginalization problem where the monopoly marks up its wholesale price over its marginal cost of production, and the downstream vendor or vendors add a second markup to the wholesale price. The monopoly can avoid the double marginalization problem by vertically integrating downstream. Alternatively, it can quasi-vertically integrate if it has two instruments. For example, it can use a two-part tariff where it charges a vendor  $T$  for the right to carry its product and a constant wholesale unit price.

Because the monopoly can use  $T$  to capture a vendor's profit, the monopoly wants its vendor to maximize this profit. If the monopoly sells to only one firm, it sets its wholesale price equal to its production cost so that the vendor charges the same price as would an integrated monopoly. If the monopoly sells to  $N$  downstream firms, it sets its wholesale price so that the resulting downstream price is the same as that of the integrated monopoly. The upstream monopoly captures all downstream profits by setting  $T$  appropriately.

Because the upstream monopoly can control the downstream price with its wholesale price regardless of the number of downstream firms, it is indifferent as to the number of vendors if there is no fixed cost (e.g., of enabling a new phone to work on a network),  $F$ , associated with

each vendor carrying its product. Given a positive fixed cost, it does not matter whether the fixed cost is paid by the upstream or downstream firm, as the upstream firm captures downstream profit and hence ultimately bears this cost. Consequently, if  $F > 0$ , the upstream monopoly wants to sell to only one downstream vendor. A MUD requirement forcing the upstream monopoly to sell to  $N$  downstream firms does not change retail price or consumer welfare, but the upstream monopoly's profit falls by  $(N - 1)F$ , and hence social welfare falls by the same amount.

In the second special case, the downstream firms price competitively. If the new and the generic goods are not substitutes and there is no fixed cost, then the upstream firm would set its wholesale price equal to the integrated monopoly price because the downstream firms do not add a markup. This result is the classic one that an upstream monopoly is indifferent between vertically integrating or not given fixed-proportions production, because the upstream monopoly can control the downstream price without integrating. With a fixed cost, the monopoly would again want only one downstream firm, and consumers are indifferent about the number of downstream vendors.

If the new and the generic products are substitutes, an integrated monopoly would act like a monopoly with respect to its residual demand curve given the competitive supply curve of the generic product. That is, the integrated firm acts like a dominant firm facing a competitive fringe. By the same reasoning as above, the upstream monopoly can quasi-integrate by setting its wholesale price so that the downstream price equals the integrated-monopoly price. If  $F = 0$ , the upstream monopoly is willing to sell to all firms at an appropriate wholesale price. However if  $F > 0$ , it uses only one downstream vendor. Again, if  $F > 0$  and MUD is imposed so that the upstream firm must use more than one downstream vendor, its profit falls as does social welfare, and consumers receive no benefit. Thus, in either of these special cases, MUD is harmful.

### 3 Substitutes

We now consider a market in which the goods are substitutes and there are a finite number of identical downstream firms. For simplicity, we assume that there is a downstream duopoly ( $N = 2$ ). We continue to assume that the downstream firms use a fixed-proportion production function and have no additional marginal cost. The monopoly's wholesale price is the sum of its cost of production plus a constant markup,  $m$ . A competitive industry with constant average costs produces the generic good, so that the downstream firms buy the generic at its cost of production.

The duopoly firms,  $i = 1$  and  $2$ , are quantity setters. The quantity  $q_{ji}$  is the amount that

Firm  $i$  sells of product  $j$ , where  $j = g$  is the generic good and  $j = n$  is the new product. For notational simplicity and to avoid the need to keep track of upstream marginal production costs, we express the price  $p_g$  for the generic and the price  $p_n$  for the new good net of their constant upstream marginal production costs.

There are four possible outcomes. The upstream monopoly may want either one or two downstream vendors, and, in either case, consumer welfare may be higher with either one or two firms. We show that even with a linear model, all four of these outcomes are possible.

### 3.1 Two-stage game

In the first stage of our two-stage game, the upstream monopoly decides how many vendors it wants and the downstream firms decide whether to accept the offer from the upstream monopoly. In the second stage, the downstream firms' actions, conditional on the number of vendors and the upstream monopoly's markup,  $m$ , result in a Cournot equilibrium and the upstream monopoly collects the transfer  $T$ .

In the first stage in the absence of a MUD requirement, the monopoly may offer a contract to a single firm (by convention, Firm 1) or to both firms. In the latter case, the monopoly offers the same contract to both firms. The outcome of the first stage determines  $m$ , the amount by which the wholesale price exceeds the upstream monopoly's marginal cost of production,  $T$ , the transfer from the vendor to the upstream monopoly, and the number of firms that have been made and accepted an offer from the monopoly to be a vendor.

In the second stage, firms choose quantities of the two goods, resulting in a Cournot equilibrium. The equilibrium values are functions of  $m$ . If Firm  $i$  is offered and accepts the monopoly's offer, its profit function is  $\pi_i = p_g q_{gi} + (p_n - m) q_{ni} - T$ , the sum of the profits from selling generics and from selling the new good minus the transfer payment. If Firm  $i$  has not received an offer from the monopoly or if it has rejected that offer, its profit function is  $\pi_i = p_g q_{gi}$ , which includes sales of only the generic good.

Let  $k$  denote the number of firms that sell the new product,  $k \in \{1, 2\}$ . The equilibrium values of  $m$  and  $T$ , denoted  $m^*(k)$  and  $T^*(k)$ , depend on  $k$ . Substituting these values into the duopoly profit functions, we write the equilibrium values of the latter as  $\pi_i^*(k)$ . When we want to denote downstream profits evaluated at a general (possibly non-equilibrium) value of  $m$ , we write  $\pi_i^*(k, m)$ .

We assume here the upstream monopoly is able to make downstream firms a take-it-or-leave-it offer, and thereby extract all the industry gains from the new product. (Section 4 discusses alternative assumptions about the ability of the monopoly to extract rent from downstream firms.) If the upstream monopoly offers the contract to only Firm 1, the monopoly sets

the licensing fee  $T$  so that in equilibrium  $\pi_1^*(1) = \pi_2^*(1) + \varepsilon$ , where  $\varepsilon \geq 0$ . That is, the net-of-transfer profit of Firm 1, which sells both goods, is greater than or equal to the profit of the firm that sells only the generic product. If  $\pi_1^*(1) < \pi_2^*(1)$ , Firm 1 would reasonably reject the offer. A sensible model includes the possibility that  $\varepsilon > 0$ , so that the license-holder does strictly better than its rival. However for now, in the interest of simplicity, we consider only the limiting case where  $\varepsilon = 0$ , i.e. where the monopoly extracts all the rent from its vendor. If one duopoly firm insisted on a value  $\varepsilon > 0$ , the monopoly could turn to the rival and obtain a slightly better deal.

Let the equilibrium level of profits when, in the absence of the upstream monopoly, both firms sell only the generic good be  $\pi^e$ . In some cases, the duopoly profits fall after the new product enters:  $\pi_1^*(k) = \pi_2^*(k) < \pi^e$ . This outcome is reasonable if each duopoly firm believes that were it to reject the monopoly's offer, the monopoly would be successful in coming to terms with its rival.

If the monopoly offers both firms a contract ( $k = 2$ ), its offer is conditional on acceptance by both firms. If one firm rejects the offer, the monopoly may alter its offer to the other firm. Given the equilibrium assumption that  $\pi_1^*(1) = \pi_2^*(1)$ , if Firm 2 rejects an offer made to both firms, it knows that its profit will be  $\pi_2^*(1)$  in the resulting equilibrium where the monopoly makes an offer to only the other firm. Because the monopoly extracts all the rent from its vendor (and can induce a firm to accept less than the ex ante level of profits), both firms receive the same level of profit in equilibrium, regardless of whether the monopoly sells to one or two firms, so that

$$\pi_1^*(1) = \pi_2^*(1) = \pi_1^*(2) = \pi_2^*(2). \quad (1)$$

We want to compare the equilibria when there are one or two vendors. We solve the upstream monopoly's problem by working backwards. First, we determine the Cournot equilibrium sales rules as functions of  $m$  when the monopoly sells to only Firm 1,  $q_{ji}^*(1, m)$ . Then we solve the upstream monopoly's problem when it sells to a single firm:

$$\Pi^*(1; F) = \max_{m, T} [mq_{n1}^*(1, m) + T - F] \text{ subject to } \pi_1^*(1, m, T) \geq \pi_2^*(1, m). \quad (2)$$

The monopoly's solution to this problem produces the equilibrium values  $m^*(1)$ ,  $T^*(1)$ ,  $\pi_i^*(1)$ , and  $\Pi^*(1; F)$ .

Using the constraint in Equation 2 and the definition of downstream profits to eliminate  $T$ , we rewrite the monopoly's maximization problem as

$$\Pi^*(1; F) = \max_m \underline{p_g q_{g1}^*(1, m) + p_n q_{n1}^*(1, m)} - F - \pi_2^*(1, m). \quad (3)$$

In a Stackelberg equilibrium, the leader maximizes its profit subject to the best-response function of the follower. In the usual Stackelberg setting, both leader and follower sell a single

product. In our setting, Firm 1 might sell both the new and the generic product, while Firm 2 sells only the generic product. However, we use the terms “Stackelberg leader and follower” in the standard way: the leader maximizes its total profit, subject to the follower’s best-response function.

Equation 3 illuminates the relation between the equilibrium in our problem and the equilibrium in which a single vendor of the new product behaves as a Stackelberg leader in the game in which both firms sell the generic and only the leader sells the new product. The downstream Stackelberg leader’s profit is the underlined term in Equation 3: the profit of the single vendor net of the markup and the transfer. The Stackelberg leader maximizes this profit. However, the upstream monopoly wants to maximize this term minus the profit of the non-vendor, because the monopoly can use the transfer to capture rents equal to the difference in the downstream firms’ profits. The monopoly benefits not only by increasing its vendor’s pre-transfer profit, but also by decreasing the non-vendor’s profit.

The level of  $m$  that induces the single-vendor to act like a Stackelberg leader (i.e., that maximizes the underlined term in Equation 3) satisfies the necessary condition

$$\frac{d [p_g q_{g1}^* (1, m) + p_n q_{n1}^* (1, m)]}{dm} = 0. \quad (4)$$

The level of  $m$  that the monopoly chooses to solve the problem in Equation 3, satisfies

$$\frac{d [p_g q_{g1}^* (1, m) + p_n q_{n1}^* (1, m)]}{dm} - \frac{d \pi_2^* (1, m)}{dm} = 0. \quad (5)$$

An increase in  $m$  causes Firm 1 to reduce sales of the new product, thereby increasing Firm 2’s profit, so  $\frac{d \pi_2^* (1, m)}{dm} > 0$ . Given this result and the assumed concavity of the monopoly’s maximand, the monopoly’s optimal choice of  $m$  is strictly less than the level of  $m$  that would induce its agent to behave as a single-vendor Stackelberg leader. As a consequence, the single vendor in this equilibrium chooses higher new product sales than would a single-vendor Stackelberg leader. Consequently, aggregate downstream profits, net of the markup and the transfer, are lower here than in the single vendor Stackelberg equilibrium.

To solve the problem when the monopoly sells to both firms, we obtain the symmetric Cournot equilibrium sales rules as functions of  $m$ ,  $q_j^* (2, m)$ . Here, we drop the firm index because the equilibrium is symmetric. We then solve the monopoly’s problem

$$\Pi^* (2; F) = \max_{m, T} 2[m q_n^* (2, m) + T - F] \text{ subject to } \pi_i^* (2, m, T) \geq \pi_2^* (1). \quad (6)$$

The value  $\pi_2^* (1)$  is the value of a downstream firm’s outside option;  $\pi_2^* (1)$  is a constant, determined by the equilibrium when the monopoly sells to a single firm, i.e. by the solution to the problem in Equation 3. This term affects the value of the monopoly’s payoff (because it affects



the transfer), but it has no effect on the optimal level of the markup when the monopoly sells to two firms.

Using the definition of profits and the constraint in Equation 6, we rewrite the monopoly's maximization problem when it uses two vendors as

$$\Pi^*(2; F) = \max_m \underline{2[p_g q_g^*(2, m) + p_n q_n^*(2, m)]} - 2(\pi_2^*(1) + F). \quad (7)$$

The underlined term in Equation 7 is aggregate downstream profit before payment of the markup and the transfer. Because  $\pi_2^*(1)$  is a constant in this problem, maximization of the right side of Equation 7 is equivalent to maximizing the underlined term in that equation. Thus, an upstream monopoly that uses two vendors chooses the markup to maximize downstream profits, exclusive of the markup and transfer.

In summary, we see that the monopoly that sells to one firm chooses a markup that leads to downstream profits (before payment of the transfer and markup) that are lower than in the Stackelberg equilibrium; in contrast, the monopoly that sells to two firms chooses a markup that maximizes downstream profits (before payment of the transfer and the markup). If only the generic were sold, consumer welfare would be higher in a Stackelberg equilibrium than in a Cournot equilibrium, and consumer welfare would be even higher if aggregate output exceeds the level in the Stackelberg equilibrium as with one vendor. This analogy to non-differentiated goods suggests that consumers may be better off when the monopoly sells to a single vendor. However, the analogy is not exact because the goods are differentiated. Hence, increasing the output of one good while decreasing the output of the other complicates the welfare comparison.

### 3.2 Linear model assumptions

We now assume that the inverse demand functions for both the new and the generic product are linear:

$$\begin{aligned} p_g &= a - b(q_{g1} + q_{g2}) - c(q_{n1} + q_{n2}), \\ p_n &= A - B(q_{n1} + q_{n2}) - C(q_{g2} + q_{g1}). \end{aligned} \quad (8)$$

The intercepts  $a$  and  $A$  equal the intercept of the inverse demand curve minus the constant marginal production cost. All parameters are non-negative. Because these linear demand equations lead to closed-form expressions for the equilibrium sales rules, we can solve for the equilibrium levels of  $m^*(1)$  and  $m^*(2)$  and then compare the price levels and consumer welfare in the two scenarios.<sup>1</sup>

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<sup>1</sup>Moner-Colonques, Sempere-Monerris, and Urbano (2004) examine a market in which two upstream firms decide whether to sell their products through one or both of the downstream vendors. Their model allows the

The model has seven parameters,  $a$ ,  $A$ ,  $b$ ,  $B$ ,  $c$ ,  $C$ , and  $F$ . By choosing the units of the quantities and the prices, we can set any three parameters, say  $a$ ,  $A$ , and  $b$ , equal to arbitrary positive numbers, leaving four free parameters,  $B$ ,  $c$ ,  $C$ , and  $F$ . In the interest of simplicity, we consider a special “almost symmetric” case where  $a = A$  and  $b = B$ , so that Equations 8 become

$$\begin{aligned} p_g &= a - b(q_{g1} + q_{g2}) - c(q_{n1} + q_{n2}), \\ p_n &= a - b(q_{n1} + q_{n2}) - C(q_{g1} + q_{g2}). \end{aligned} \quad (9)$$

That is, the intercepts and own-quantity slopes of the two products are identical up to a scaling factor. However, the cross-quantity effects, the degree to which one good substitutes for the other, differ. Combining the almost symmetry assumption with our ability to choose units allows us to arbitrarily set the values of four of the model parameters, leaving only  $c$ ,  $C$ , and  $F$  undetermined. We now concentrate on the role of these three parameters. This special case allows for all four possible outcomes, where the monopoly wants to sell to one firm or two firms, and monopoly and consumer interests are aligned or opposed.

We can rearrange Equations 9 to write the demand system as

$$\begin{pmatrix} \mathbf{q}_g \\ \mathbf{q}_n \end{pmatrix} = \begin{pmatrix} \frac{ba-ac}{b^2-cC} \\ \frac{ab-aC}{b^2-cC} \end{pmatrix} - \begin{bmatrix} \frac{b}{b^2-cC} & -\frac{c}{b^2-cC} \\ -\frac{C}{b^2-cC} & \frac{b}{b^2-cC} \end{bmatrix} \begin{pmatrix} p_g \\ p_n \end{pmatrix}. \quad (10)$$

where aggregate quantities,  $\mathbf{q}_j = q_{j1} + q_{j2}$ , are functions of prices. Because the two goods are substitutes, we want the aggregate quantity demanded of a good to decrease as its own-price increases and rise with respect to the other price. In addition, we require that the aggregate demand for both goods be positive if both prices, which are net of their production costs, are zero:  $p_g = p_n = 0$ . These two conditions imply the parametric restrictions that the own-quantity parameter is greater than either of the other-quantity parameters:

$$b > c, \quad b > C. \quad (11)$$

That is, an increase in the own-quantity has a larger effect on price than a comparable increase of the other-quantity.

These linear demand functions are approximations to a more general system of demand functions.<sup>2</sup> The relative magnitude of  $c$  and  $C$  plays a major role in the analysis. We use the indirect utility function to provide an economic interpretation of the sign of  $c - C$ . Denoting

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upstream firms to charge only a per unit price, whereas our upstream firm also uses a transfer. In addition, we allow the cross-price coefficients  $c$  and  $C$  to differ. The difference in these coefficients is key to our results.

<sup>2</sup>The quadratic utility function produces linear demand functions. However, symmetry of the cross partials of the Hicksian demand functions requires  $c = C$  (Singh and Vives, 1984).

$\mathbf{q}_j^{\text{Hicksian}}$  as the Hicksian demand and  $\mathbf{q}_j$  as the Marshallian demand (as above), the Slutsky equation is

$$\frac{\partial \mathbf{q}_j}{\partial p_i} = \frac{\partial \mathbf{q}_j^{\text{Hicksian}}}{\partial p_i} - \mathbf{q}_i \frac{\partial \mathbf{q}_j}{\partial y}.$$

We obtain the partial derivatives,  $\frac{\partial \mathbf{q}_i}{\partial p_i}$ , from Equations 10. We then use the the symmetry relation

$$\frac{\partial \mathbf{q}_g^{\text{Hicksian}}}{\partial p_n} = \frac{\partial \mathbf{q}_n^{\text{Hicksian}}}{\partial p_g},$$

and the Slutsky equation to show that the parameters of the demand equation satisfy

$$\frac{\partial \mathbf{q}_g}{\partial p_n} - \frac{\partial \mathbf{q}_n}{\partial p_g} = \frac{c - C}{b^2 - cC} = \frac{\mathbf{q}_g \mathbf{q}_n}{y} (\eta_n - \eta_g), \quad (12)$$

where  $\eta_j$  is the income elasticity of demand for commodity  $j$ . The important result is that the sign of  $c - C$  is the same as that of the difference between the income elasticities,  $\eta_n - \eta_g$ , because  $b^2 - cC$  must be positive by Inequalities 11. If, as seems reasonable, consumers view a new product as more of a luxury than a generic product so that the income elasticity of the new product is greater than that of the generic, then  $c > C$ .

### 3.3 Equilibria

To compare the equilibrium with linear inverse demand functions where the monopoly sells to two firms to the equilibrium where it sells to only one firm, we first determine the downstream firms' Cournot equilibrium quantities given  $m$ . We then use that information to solve the upstream monopoly's profit maximizing problem to determine  $m$ .

Using the inverse demand Equations 8, the profit of downstream Firm  $i$ , exclusive of any transfer is

$$\pi_i = [a - b\mathbf{q}_g - c\mathbf{q}_n]q_{gi} + [a - b\mathbf{q}_n - C\mathbf{q}_g - m]q_{ni}. \quad (13)$$

Because Firm 1 always sells both goods, its first-order conditions are

$$\frac{\partial \pi_1}{\partial q_{g1}} = a - 2bq_{g1} - bq_{g2} - c\mathbf{q}_n - Cq_{n1} = 0, \quad (14)$$

$$\frac{\partial \pi_1}{\partial q_{n1}} = a - 2bq_{n1} - bq_{n2} - C\mathbf{q}_g - cq_{g1} = 0. \quad (15)$$

If Firm 2 does not sell the new product,  $q_{n2} = 0$  in Equation 15, and hence  $\mathbf{q}_n = q_{n1}$  in Equation 14.

If Firm 2 sells both goods, then its first-order conditions are the same as Equations 14 and 15 with the subscripts 1 and 2 reversed. However, if Firm 2 sells only the generic good, it has a single first-order condition for the generic good:

$$\frac{\partial \pi_2}{\partial q_{g2}} = a - 2bq_{g2} - bq_{g1} - cq_{n1} = 0. \quad (16)$$

If the monopoly sells to both firms, there are four first-order conditions: Equations 14 and 15 and the same pair of equations with the subscripts 1 and 2 reversed. However, given symmetry, these four equations collapse into two first-order conditions, where the quantity for each firm is replaced by  $q_j/2$ . Given that the monopoly sells to both firms, it is indifferent about the division of sales between the two firms. It has one instrument,  $m$ , to influence two targets: aggregate new product and aggregate generic quantities,  $q_g$  and  $q_n$ .

In contrast, if the monopoly sells to only Firm 1, then there are three relevant first-order conditions, Equations 14, 15, and 16. Thus when the monopoly sells to a single firm, it has the same single instrument to affect three targets: new product sales by Firm 1, aggregate generic sales, and Firm 1's share of generic sales (or equivalently,  $q_{n1}$ ,  $q_{g1}$ , and  $q_{g2}$ ). For a given level of aggregate generic sales, the monopoly prefers its agent, Firm 1, to have a larger share so as to increase Firm 1's pre-transfer profit, which the monopoly captures through the transfer  $T$ .

When the monopoly sells to only Firm 1, we can solve the first-order conditions, Equations 14 – 16, to obtain the duopoly sales as functions of  $m$ . Differentiating these expressions with respect to  $m$  and using the parameter restrictions in Equation 11, we obtain the comparative statics results:

$$\frac{dq_{n1}}{dm} < 0 < \frac{dq_{g1}}{dm}. \quad (17)$$

(This inequality also holds when the monopoly sells to two firms.) An increase in  $m$  reduces Firm 1's marginal profit from each new product sale, causing it to reduce sales in that market. That reduction in new product sales causes Firm 1's marginal revenue curve in the generic market to shift out, increasing sales in that market. The equilibrium level of Firm 2 generic sales (when the monopoly sells only to Firm 1) is

$$q_{g2} = -\frac{(C - c)}{(6b^2 - 4cC - C^2 - c^2)}m + \text{a constant}. \quad (18)$$

Consequently,

$$\frac{dq_{g2}}{dm} \begin{cases} < \\ = \\ > \end{cases} 0 \text{ for } \begin{cases} C > c \\ C = c \\ C < c \end{cases}.$$

The value of  $m$  has only an indirect effect on Firm 2's profit due to the changes in Firm 1 sales, described in Inequality 17. The changes in Firm 1's sales of the two goods, resulting from a change in  $m$ , have counteracting effects on Firm 2's marginal revenue curve. As  $m$  increases, Firm 1 sells fewer units of the new good, which helps Firm 2, but more units of the generic product, which hurts Firm 2. Given our discussion of Equation 12, it is reasonable to expect that  $C < c$ , so  $dq_{g2}/dm > 0$ .

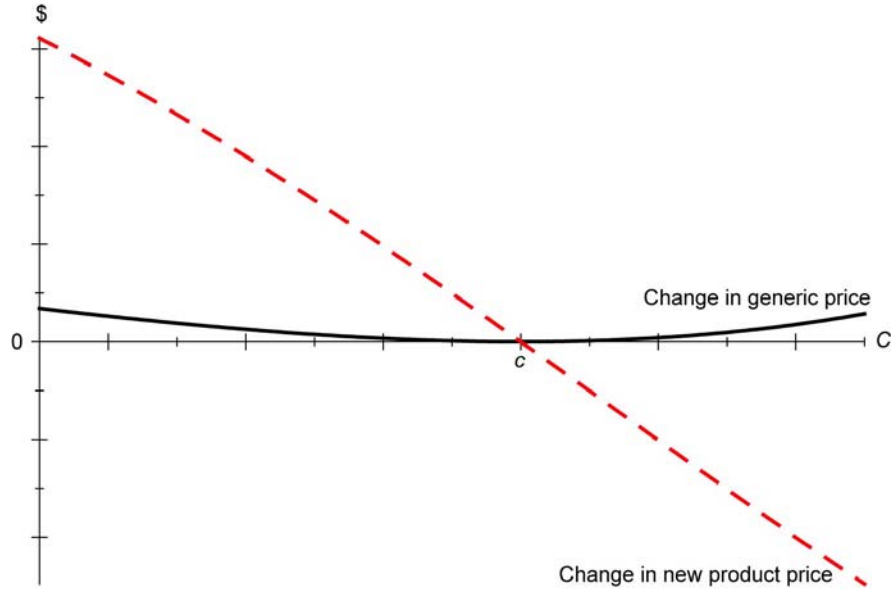


Figure 1: Changes in prices from adding a second vendor.

An increase in  $m$  affects only the term  $bq_{g1} + cq_{n1}$  within Firm 2's marginal revenue, Equation 16. Straightforward calculations show that

$$\frac{d(bq_{g1} + cq_{n1})}{dm} = 2b \frac{C - c}{6b^2 - 4Cc - C^2 - c^2}.$$

The denominator of the right-hand-side term is positive by Inequalities 11. Thus, an increase in  $m$  shifts up Firm 2's marginal revenue curve if and only if  $C < c$  (so the numerator of the right-hand-side term is negative). If Firm 2's marginal revenue shifts up, it chooses to sell more units of the generic good.

If  $C = c$ , then a change in  $m$  does not affect the marginal revenue curve, so  $q_{g2}$  is a constant. Here, the monopoly has two targets – the same number as when the monopoly sells to both firms. However, if  $C \neq c$ , when the upstream monopoly sells to a single firm, it realizes that its choice of  $m$  affects the equilibrium choice of Firm 2's generic sales; it then has three targets compared to two when it sells to both firms. These comparative statics results help to explain the relation between parameter values and the manner in which the choice of one vendor or two vendors affects prices.

The ability to obtain explicit formulae for the equilibrium decision rules enables us to compare the new product and the generic prices in the two regimes (Appendix A.1). If the upstream monopoly sells to two firms rather than one, we show that the generic price is higher for  $c \neq C$  and is unchanged for  $c = C$ , while the new product price is higher for  $C < c$ , equal for  $c = C$ , and smaller for  $C > c$ .

Although we obtain all our main results analytically, we illustrate the more important ones using simulations.<sup>3</sup> Figure 1 shows how the generic and new product prices change, as a function of  $C$ , if the upstream monopoly adds a second vendor (where we fix  $c, F > 0$ , and the other parameters). When two firms sell the new product, each firm internalizes some portion of the effect of generic sales on the new-product price. As a result, the generic price increases (when two rather than one firm sells the new product), unless  $C = c$ , when it is unchanged. Having two firms sell the new product increases the price of the new product for  $C < c$ , and decreases the price for  $C > c$ . By Equation 12, if the income elasticity for the new good is greater than that of the generic, then  $c > C$ , so the prices of both goods rise as the number of downstream vendors increases from one to two.

### 3.4 Welfare

By Equation 1, the downstream firms' profits are the same regardless of whether the monopoly uses one vendor or two. Therefore, the total welfare effect of a MUD requirement depends on only its effects on consumer welfare and monopoly profit. We first consider monopoly profits and then consumer welfare.

For  $F = 0$  the monopoly strictly prefers to sell to two vendors for  $c \neq C$  and is indifferent between selling to one vendor or two vendors if  $c = C$ . For  $F > 0$  the monopoly prefers to sell to a single vendor if and only if  $|c - C|$  is small. To demonstrate this claim, we examine the two cases, where the upstream monopoly sells to one firm and where it sells to two firms. In both cases we use the firms' necessary conditions to write their sales as functions of  $m$ . For each of the two cases, we substitute these sales rules into the monopoly's profit functions, given by Equations 2 and 6. Each of these profit functions is quadratic in  $m$ . We maximize each function with respect to  $m$  to obtain the equilibrium monopoly profits in the two cases,  $\Pi^*(1; 0)$  and  $\Pi^*(2; 0)$ . Subtracting the former from the latter, we find that

$$\Pi^*(2; 0) - \Pi^*(1; 0) = \frac{1}{9}a^2(b + C)^2(c - C)^2 \frac{(c - 2b + C)^2}{b(Cc - b^2)^2(9b^2 - 2c^2 - 2C^2 - 5Cc)} - F. \quad (19)$$

The denominator of the last factor on the right side of this equation is positive by Inequality 11. When  $F = 0$ , the difference in profits is therefore positive for  $c \neq C$  and zero for  $c = C$ . In addition, for  $F > 0$ , in the neighborhood where  $c \approx C$ ,  $\Pi^*(2; 0) - \Pi^*(1; 0) < 0$ . Therefore, for a given  $|c - C|$ , the larger  $F$ , the "more likely" that the upstream monopoly prefers to sell

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<sup>3</sup>Given our earlier assumptions, we have three free parameters,  $c, C$ , and  $F$ , but only  $c$  and  $C$  have a direct effect on the equilibrium quantities for a given number of downstream vendors. We set  $a = 10$  and  $b = 10/9$  in all our simulations. Given Inequalities 11, these parameter choices imply that  $c$  and  $C$  must each be less than  $10/9$ . For specificity, we set  $c = 0.5$  and  $F = 0.2$ , and examine how the results vary with  $C$ .

to a single firm. All else the same, using two vendors rather than one reduces the monopoly's profit by  $F$ .

We can use these results to examine consumer welfare. For  $C < c$ , both the new product and the generic prices are higher when the monopoly uses two vendors rather than a single vendor; therefore, consumers prefer a single vendor if  $C < c$ . For  $C > c$  adding a second vendor increases the generic price and decreases the new product price, so the effect on consumer welfare of the second vendor is ambiguous in general. However, for  $C > c$  and  $C - c$  "sufficiently small," consumer welfare is higher when the monopoly sells to two vendors. Appendix B provides a formal statement and proof of this claim, but the intuition is clear from Figure 1. The generic price under two vendors minus the generic price with one vendor is minimized at  $C = c$ , where the price difference is zero. Therefore, in the neighborhood of  $C = c$ , this price difference is of second order in  $C - c$  (i.e., the first-order Taylor expansion, with respect to  $C$ , of the price difference, evaluated at  $C = c$  is zero).<sup>4</sup> Consequently, the loss in consumer welfare arising from the higher generic price (when the monopoly moves from one vendor to two vendors) is a second-order effect. However, as is evident from Figure 1, adding a second vendor creates a first-order decrease in the new product price, and therefore creates a first-order welfare gain for  $C > c$ . Therefore, the first-order approximation of the change in consumer welfare (evaluated at  $C = c$ ) from the addition of a second vendor is positive.

Figure 2 illustrates the effect on the monopoly's profit and consumer welfare of adding a second vendor, given that  $F > 0$ .<sup>5</sup> The dashed curve is the change in a representative consumer's utility from having two rather than one vendor of the new product. That this curve crosses the axis at  $C = c$  from below follows from our discussion of consumer welfare. This curve is independent of  $F$ . The solid curve is the change in the monopoly's profit from having two downstream vendors rather than one. It illustrates Equation 19, which shows that the change in monopoly profit from selling to two firms rather than to one is negative in the neighborhood of  $C = c$  for  $F > 0$ , but is positive where  $|c - C|$  is large relative to  $F$ .

Figure 2 shows that, given parameters values  $a$ ,  $b$ ,  $c$ , and for  $F > 0$ , there is a value  $e < c$  such that (i) for  $C < e$  the monopoly prefers to sell to two firms, but consumers are better off when the monopoly sells to a single firm; (ii) for  $e < C < c$  both consumers and the monopoly are better off when the monopoly sells to a single firm; (iii) for  $C > c$  with  $C - c$  sufficiently small, consumers are better off when the monopoly sells to two firms, but the monopoly prefers

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<sup>4</sup>This claim can be verified immediately using the equation for the difference in generic prices, Equation 22 in the Appendix. Similarly, the claim regarding the difference in new product price can be verified by using Equation 23.

<sup>5</sup>We use the same parameters as above to simulate this figure. This figure uses the *change in consumer surplus* only to illustrate the change in consumer welfare. Neither our heuristic argument in the text nor the formal statement and proof in Appendix B concerning the change in consumer welfare involve consumer surplus.

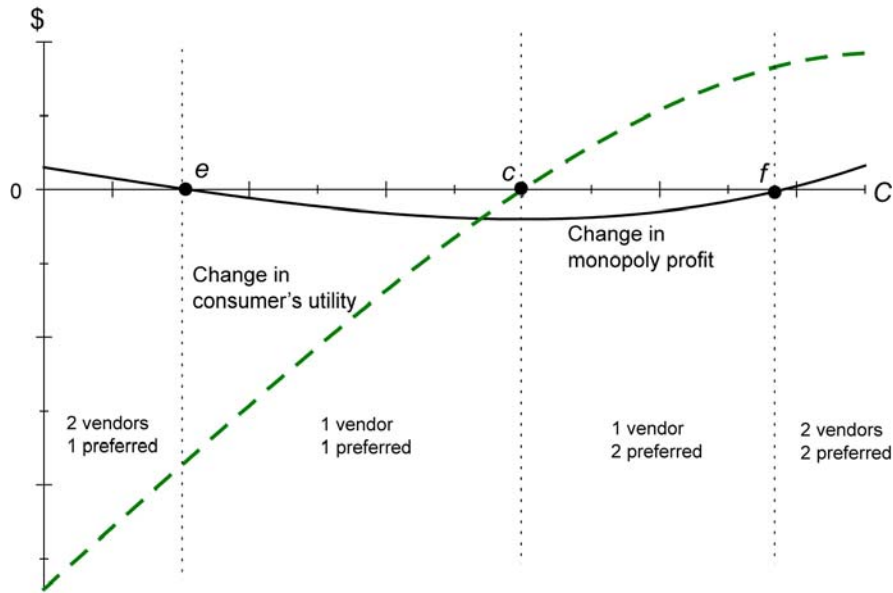


Figure 2: Change in Monopoly Profit and Consumer's Utility from Adding a Second Vendor

to sell to a single firm. In addition, (iv) for sufficiently small  $F$  there is a value  $f > c$  such that for  $C > f$  with  $C - f$  small, the monopoly prefers to sell to two firms and consumers also prefer that the monopoly sells to two firms.<sup>6</sup>

Claims (i) – (iii) follow immediately from inspection of Figure 2 and from the preceding discussion on the monopoly's profit and consumer welfare. To demonstrate (iv), we denote  $\Gamma$  as the closure of the set of  $C > c$  at which consumers prefer that the monopoly use two vendors. From the comments on consumer welfare,  $\Gamma$  has positive measure. Because we can make  $f$  arbitrarily close to  $c$  by choosing  $F > 0$  but small, we can insure that  $f \in \Gamma$  for small  $F > 0$ . Such a value of  $f$  corresponds to the value shown in Figure 2.

We have seen that adding a second vendor increases the new-product price when  $C < c$  and decreases that price when  $C > c$ . At a given markup, the increased competition arising from the presence of a second vendor tends to decrease the new-product price. However, the monopoly adjusts the markup when it adds a second vendor. Figure 3 shows the equilibrium markup,  $m$ , with one or two downstream vendors and Appendix A.2 shows that these qualitative features hold generally for the linear model.

The monopoly subsidizes a single vendor ( $m < 0$ ) if  $C < c$ , where generic sales have a relatively small effect on the new-product price. The monopoly uses the subsidy to assist its agent in increasing its share of the generic sales. The monopoly then extracts its agent's profit

<sup>6</sup>We do *not* assert that for all  $C > f$  consumers prefer two vendors. Claim (iv) is limited to the neighborhood of  $f$  for small  $F$ .



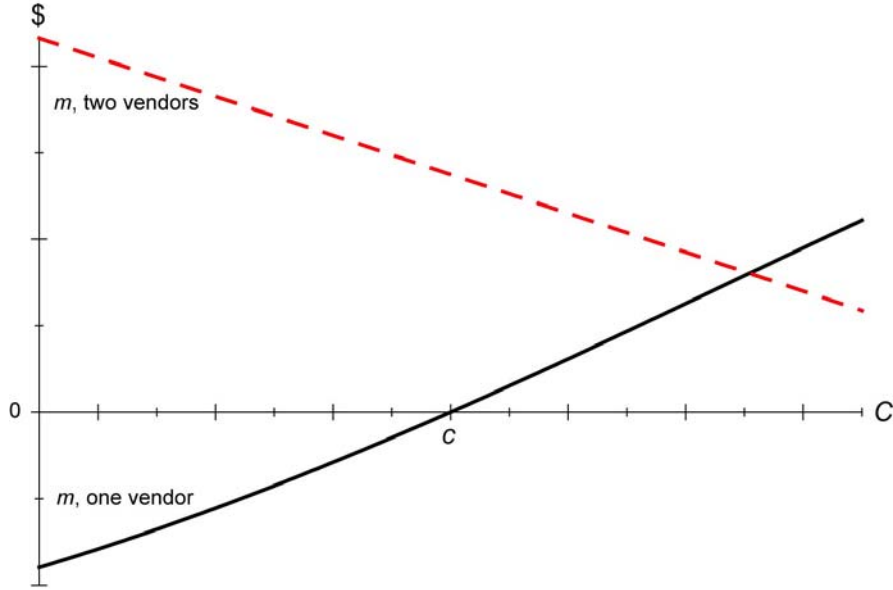


Figure 3: The upstream monopoly's markup with one or two vendors.

from generic sales using the licensing fee,  $T$ . The monopoly uses a positive markup when it sells to a single firm and  $C > c$ . The monopoly always uses a positive markup when it sells to two firms. As  $C$  increases, the new-product price becomes more sensitive to generic sales; the equilibrium markup rises with  $C$  if the monopoly sells to a single firm and decreases if it sells to two firms. Figure 3, by illustrating how  $m$  varies with respect to  $C$  for one or two vendors, helps explain why selling to a second vendor increases the new-product price when  $C < c$  and decreases that price for  $C > c$ , as Figure 1 shows. For example, when  $C < c$ ,  $m$  is negative with one vendor and positive with two vendors, so adding a second vendors raises the price of the new good.

### 3.4.1 The effect of monopoly entry

These last several results compare the one-vendor and two-vendor equilibria, conditional on monopoly entry. We now turn to an analysis of the effect of monopoly entry.

The downstream firms have the same level of profits regardless of whether the upstream monopoly sells to one or to both firms. We calculate this profit level and subtract the equilibrium duopoly profits prior to entry of the upstream monopoly. This difference is

$$\left[ \frac{1}{36} a^2 (b - C) \frac{4b^2 - cb - 3Cc + C(b - C)}{(Cc - b^2)^2 b} \right] (C - c).$$

The term in square brackets is always positive, so the sign of the expression equals the sign of  $C - c$ . For  $C < c$ , the monopoly extracts some of the pre-entry oligopoly rent, in addition to all

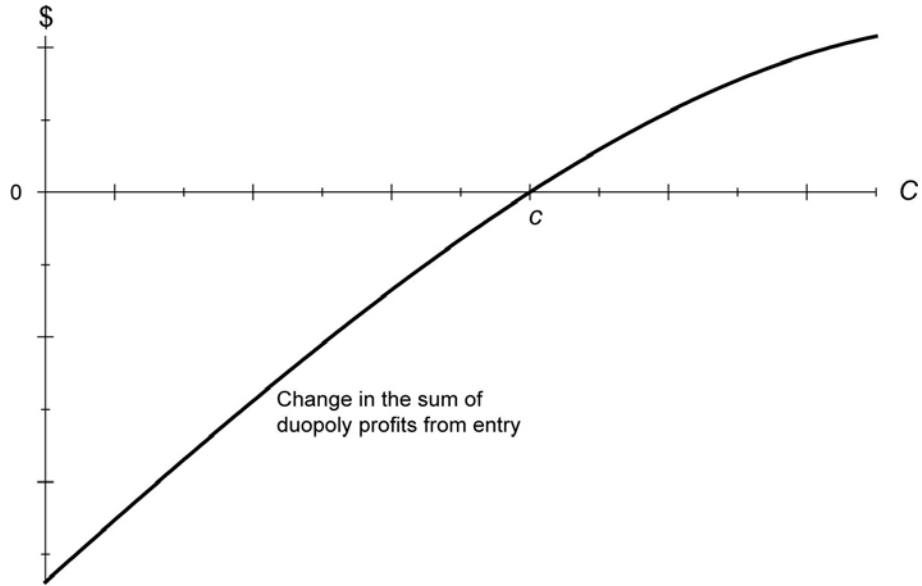


Figure 4: Change in the sum of the downstream profits from the entry of the upstream monopoly.

of the extra rent that arises from the new product. For  $C > c$ , the downstream firms obtain some of this additional rent. Figure 4 illustrates the effect of monopoly entry on duopoly profits.

That monopoly entry reduces duopoly profits is consistent with the comparative statics of the equilibrium markup. We noted that the monopoly uses a subsidy when it sells to a single vendor given that  $C < c$ . This subsidy causes the non-vendor, Firm 2, to face strong competition and it erodes that firm's profits. Our assumption about the first-stage equilibrium means that the monopoly uses the transfer to drive profits of its vendor(s) to the level received by a non-vendor in the one-vendor regime.

We can show analytically that, for  $C < c$ , entry by the new-product monopoly increases consumer welfare, regardless of whether the monopoly sells to one firm or two.<sup>7</sup> Simulations indicate that even for  $C > c$  consumer welfare, approximated by consumer surplus, is higher when the upstream monopoly enters the market. That is, consumers benefit from the new product.

## 4 Alternative equilibria

Our objective has been to analyze the consequences of MUD when the upstream monopoly has almost all of the bargaining power. We did not propose a specific bargaining game that leads

<sup>7</sup>This result follows immediately from Lemma 2 in Appendix B and the fact that  $\mathbf{q}_n = 0$  before the monopoly enters.

to the outcome that we employed in the first stage of our model. That is, we did not consider the timing of moves and the beliefs and the outside options that make the two optimization problems in Equations 3 and 7 the “right” problems for determining the equilibrium to the first stage. Although construction of that bargaining game might be an interesting enterprise, our goal is more limited.

First, we discuss our assumption in Section 3.3 that, at the bargaining stage, a downstream firm accepts a contract that gives it at least  $\varepsilon$  more than the amount that would be received by a non-vendor, even if its profit will be lower than the ex ante level. We then consider the obvious alternative to this assumption: downstream firms accept only offers that give them at least  $\varepsilon$  more than the non-vendor’s level in the one-vendor equilibrium *and at least  $\varepsilon$  more than the ex ante level*. We obtain two major results. The first is that the principal welfare comparisons are unchanged from our previous model. The second is that, despite its apparent plausibility, the alternative assumption (unlike the assumption adopted in Section 3.3) is inconsistent with profit-maximizing behavior.

## 4.1 Normal-form bargaining game

Section 3.4.1 notes that entry by the upstream monopoly reduces equilibrium downstream profits if  $C < c$ . In a second-best setting, the possibility that a new opportunity reduces an equilibrium payoff is not unusual. However, in our setting, the reduction of downstream profits due to upstream entry might appear odd, because the downstream firms might have an alternative that leads to a better outcome.

To illustrate this point, we consider a normal-form game in which the upstream monopoly offers both downstream firms contracts  $(m, T)$  that are conditioned on whether one firm or both firms accept the offer. Each firm then simultaneously chooses the action  $x \in \{A, R\}$ , where  $A$  indicates that the firm accepts the offer, and  $R$  indicates rejection. If both downstream firms reject the contract, the outcome is the ex ante (prior to monopoly entry) Cournot equilibrium. This simultaneous move game eliminates all aspects of bargaining. Despite its limitations as a model of the interaction between the upstream monopoly and the downstream firms, it provides a useful starting point for discussing the possibility that downstream firms might be harmed by the presence of the upstream monopoly. For the remainder of this section, we consider only circumstances where that result occurs, such as where  $C < c$  in our “almost symmetric” linear model.

In the absence of any constraint on the contingent contracts, the monopoly could offer a menu such that if Firm  $i$  accepts and Firm  $j$  rejects, Firm  $j$ ’s profit is extremely small and Firm

$i$ 's profit is large. In this way, the monopoly can ensure that  $x = A$  is a risk-dominant strategy.<sup>8</sup> To ensure the risk dominance of  $x = A$ , the monopoly might have to accept a very low profit in the non-equilibrium outcome where one firm accepts and the other rejects. Such a contract gives the monopoly a considerable (perhaps implausible) ability to make commitments about actions at non-equilibrium outcomes.

We can limit the monopoly's power by requiring that the contract, contingent on acceptance by one firm and rejection by the other, maximizes the monopoly's profit subject to the constraint that the agent who accepted receive a profit  $\varepsilon$  greater than the profit of the firm that rejected the contract. For  $\varepsilon = 0$ , we obtain a normal-form game analogy to our model.

If both firms reject the contingent contracts, each obtains the ex ante duopoly profit, which we normalize to be zero. Because we are interested in determining the circumstances where monopoly entry lowers downstream profits, the payoffs when at least one firm accepts the offer are all negative. Suppose that these payoffs are those in Table 1. The vendor does better than the non-vendor by the amount  $\varepsilon$ , and if both accept, they share  $\varepsilon$ . For  $0 < \varepsilon < 1$  there are two pure strategy equilibria,  $(R, R)$  and  $(A, A)$  and a symmetric mixed strategy equilibrium,  $\Pr(x = R) = \varepsilon/(2 - \varepsilon)$ , a value that ranges from 0 to 0.5 as  $\varepsilon$  ranges from 0 to 1. The outcome  $(R, R)$  is Pareto dominant and it is risk dominant for  $\varepsilon < \frac{2}{3}$ .

	$R$	$A$
$R$	$0, 0$	$-1, -1 + \varepsilon$
$A$	$-1 + \varepsilon, -1$	$-1 + \frac{\varepsilon}{2}, -1 + \frac{\varepsilon}{2}$

Table 1: Payoffs in the simultaneous move game

This normal-form game illustrates the multiplicity of equilibria; it also shows that, in a symmetric mixed-strategy equilibrium when  $\varepsilon$  is small, firms are more likely to accept than to reject the offer, even when the Pareto dominant and risk dominant actions are to reject. Thus, even in this very simple model, the equilibrium where both firms are worse off may occur.

## 4.2 Alternative reduced-form bargaining game

We now consider an alternative to the reduced-form bargaining game described in Section 3.3. This alternative eliminates the possibility that monopoly entry lowers downstream profits. However, it leads to similar welfare results as in the game where duopoly profits do fall. In addition, the alternative may be less plausible than our original model.

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<sup>8</sup>In a symmetric two-player game with two strategies per player, an action is risk-dominant if both players prefer that action when they think that there is a 0.5 probability that the other agent will choose that action.

In this alternative, we assume that all agents believe that each downstream firm will accept an offer if and only if its equilibrium profit under that offer is at least as large as its ex ante profit and is also at least as large as the profit of the rival that did not receive an offer. Given these beliefs, when the monopoly makes an offer to only Firm 1, its problem is

$$\hat{\Pi}(1; F) = \max_{m, T} [mq_{n1}^*(1, m) + T - F] \text{ subject to } \pi_1^*(1, m, T) \geq \max\{\pi_2^*(1, m), \pi^e\}. \quad (20)$$

For our linear model, the binding constraint is  $\pi_1^*(1, m, T) \geq \pi^e$  when  $C < c$  and  $\pi_1^*(1, m, T) \geq \pi_2^*(1, m)$  when  $C > c$ . That is, the constraint that the agent receive a profit that is no smaller than the ex ante profit changes the problem only for  $C < c$ . We restrict attention to this part of parameter space, because we are interested in situations where the alternative assumption about bargaining affects the outcome. We consider the effect of the new constraint on the equilibrium outcome, and we then discuss the plausibility of the new constraint.

Using the constraint on the level of the agent's profits ( $\pi_1^*(1, m, T) \geq \pi^e$  for  $C < c$ ) to eliminate the transfer, the monopoly's problem when it sells to a single firm is

$$\hat{\Pi}(1; F) = \max_m \underline{p_g q_{g1}^*(1, m) + p_n q_{n1}^*(1, m)} - F - \pi^e. \quad (21)$$

In contrast to the problem in Equation 3, the final term in the maximand in Equation 21 is a constant. Here, the monopoly chooses  $m$  to induce its vendor to sell at the level of the Stackelberg leader. In contrast, under our earlier assumption (Section 3.1) the monopoly induces its vendor to behave more aggressively so as to reduce the non-vendor's profit and thereby increase the transfer it extracts from its agent. Consequently, when the monopoly sells to a single vendor, consumers are better off under the original equilibrium assumption, than under this alternative assumption. Where the monopoly sells to two vendors,  $\pi^e$  replaces the last term,  $\pi_2^*(1)$ , in the maximand in Equation 7. However, as we noted in our discussion of Equation 7, this constant does not affect the equilibrium value of  $m$  and therefore does not affect consumer welfare. That is, this alternative assumption decreases consumer welfare when the monopoly sells to a single vendor and does not change consumer welfare when the monopoly sells to two vendors. Nevertheless, for the linear model we can show (Appendix A.3) that if the monopoly is constrained to give its agents profits that are at least equal to the ex ante profits, consumers prefer the monopoly to sell to a single vendor rather than to two vendors when  $C < c$ .

Simulations show that under the alternative assumption about the bargaining equilibrium, the upstream monopoly strictly prefers to sell to two agents when  $C < c$  for  $F$  sufficiently small. For larger values of  $F$ , the monopoly prefers to sell to a single agent. Thus, even under the alternative assumption about the allocation of industry rent, the monopoly may prefer to sell to either one or two vendors, while consumers always prefer a single vendor. The downstream firms are indifferent, because their profits are fixed at the ex ante level in both cases. Thus, our major results are robust to assumptions about the equilibrium to the bargaining game.

We now consider the plausibility of this alternative assumption about bargaining. The basis for this assumption is that all agents in the model believe that no downstream firm would accept an offer that gives it a profit that is smaller than the ex ante level. However, these beliefs are, in general, not consistent with individually optimal actions. Suppose, for example, that if the upstream monopoly decides to use a single vendor and if Firm 1 rejects a contract, the monopoly will then offer a contract to Firm 2 sufficiently attractive to induce Firm 2 to accept. We find that in the linear model with  $C < c$ , the belief that both downstream firms will reject all contracts giving them profits less than the ex ante level is not consistent with profit-maximizing behavior (Appendix A.4). Therefore, this belief cannot be used to support an equilibrium in which the monopoly makes offers that give its agents profits that are no less than the ex ante level.

Although our proof of this claim relies on the linear demand system, inspection of Equation 21 provides the intuition for this result. Suppose, contrary to the claim, that it is optimal for both firms to reject contracts giving them profits below their ex ante levels. Under this hypothesis, if the monopoly offers Firm 1 a contract giving it a profit below the ex ante level, Firm 1 should reject the offer. If it does reject, then the monopoly turns to Firm 2. Given beliefs that downstream firms reject offers leading to profits below the ex ante level, the monopoly chooses  $m$  to solve the problem in Equation 21 (with the subscript 2 replacing the subscript 1) and chooses

$$T = p_g q_{g2}^*(1, m) + (p_n - m) q_{n2}^*(1, m) - \pi^e,$$

so that the vendor receives the ex ante level of profit. Here, Firm 2's profit, exclusive of the markup  $m$  and the transfer  $T$ , equals the profit of a single-vendor Stackelberg leader (the underlined term in Equation 21). Firm 1 receives the Stackelberg follower profit in this game. In a *single product* market, the profit of a Stackelberg follower is lower than its profit in a Cournot equilibrium. Our situation is more complicated, because we are comparing the profit of a Stackelberg follower that sells a single product when the leader sells two products, and the profit in a Cournot equilibrium where both firms sell a single product. Due to this complication, we rely on linearity to compare profits in the two cases. This comparison shows that the Stackelberg follower that sells a single product, when the leader sells both products, receives a lower profit than in the Cournot equilibrium when both firms sell a single product. Consequently, if the upstream monopoly offers Firm 1 a contract that gives it a profit that is strictly higher than that of the Stackelberg follower but lower than the ex ante profit, Firm 1 should accept the contract. This result contradicts the hypothesis that it is optimal for both firms to reject all contracts giving them less profit than the ex ante level. Therefore, the belief that all firms will reject such contracts is not consistent with equilibrium.

In contrast, our earlier equilibrium assumption — that downstream firms will accept offers

that give them at least  $\varepsilon$  more than the outsider — is consistent with profit-maximizing behavior. If, for example, Firm 1 believes that Firm 2 will accept such offers, then it is optimal for Firm 1 to accept such an offer.

In summary, this section examined our assumption that downstream firms will accept offers that give them at least  $\varepsilon$  more than the outsider even if in the resulting equilibrium they have profits lower than the ex ante level. We made two basic points. First, replacing this assumption with the alternative that downstream firms will accept only those offers that give them at least  $\varepsilon$  more than the maximum of the outsider's profits and the ex ante profits, does not alter our principal welfare results. Second, despite the apparent plausibility of this alternative assumption, the belief that firms will behave according to it, is not consistent with profit-maximizing behavior.

## 5 Conclusions

We consider an industry where upstream firms sell to final consumers using downstream firms. Upstream, a competitive industry produces a generic product, and a monopoly produces a new good (such as Apple's iPhone). The upstream monopoly may use one or more downstream vendors. In most of our models, we assume that the downstream market is oligopolistic with quantity-setting firms. The firms engage in a two-stage game. In the first stage, the upstream firm decides how many vendors it wants and the selected downstream firms decide whether to accept the monopoly's offer.

Mandatory universal distribution (MUD) may or may not benefit consumers. If the new product and the generic are not substitutes, MUD does not help consumers, harms the upstream monopoly, and therefore lowers the sum of consumer and producer welfare. If the downstream industry has many firms that are price takers, we get the same results regardless of whether the products are substitutes.

If the products are imperfect substitutes and there is a downstream duopoly, there are four possible outcomes. The monopoly may want one vendor or two vendors and consumers might prefer either the monopoly's choice or the alternative. If the new product has a higher income elasticity of demand than the generic, then consumers always prefer a single vendor in our linear model. Thus, although we have identified cases where consumers might benefit from requiring the upstream firm to use a second vendor, those cases are unlikely because they require that the new product have a lower income elasticity of demand, compared to the generic. If MUD is desirable, it must be for a reason that we have not considered, such as the claim that having multiple vendors induces desirable differentiation and innovation.

## A Appendix Proofs

This appendix proves the assertions made in the text. A separate appendix, intended for Referees, provides details on the results for consumer welfare

### A.1 Comparison of downstream prices

We consider the two cases, where the monopoly sells to a single vendor or two vendors. In both cases, we substitute the equilibrium markup, obtained from maximizing the monopoly's profits, into the firms' equilibrium decision rules. We then substitute these rules into the inverse demand functions to obtain the equilibrium generic and the new product prices when the monopoly sells to one or two firms. These prices are:

$$\begin{aligned}
 \text{generic, one firm} & : \tau = \frac{1}{6} \frac{2ab^2 - cab + Cab - cCa - aC^2}{-Cc + b^2} \\
 \text{new product, one firm} & : v = -\frac{1}{6} \frac{1 - 3ab^3 + b^2Ca - abC^2 + 4abCc - 2cC^2a + aC^3}{b(-Cc + b^2)} \\
 \text{generic, two firms} & : \delta = \frac{3}{2} \frac{2ab^2 - cab + Cab - cCa - aC^2}{9b^2 - 2c^2 - 2C^2 - 5Cc} \\
 \text{new product, two firms} & : \epsilon = \frac{1}{2} \frac{9ab^3 - 4abc^2 - abC^2 - 4abCc - 7b^2Ca + 4b^2ca + 2cC^2a + aC^3}{b(9b^2 - 2c^2 - 2C^2 - 5Cc)}.
 \end{aligned}$$

For both generic and new product prices, we subtract the equilibrium price with two firms from the equilibrium price with a single firm:

$$\delta - \tau = \frac{1}{3} a (c - C)^2 (b + C) \frac{2b - c - C}{(-cC + b^2)(-5cC + 9b^2 - 2c^2 - 2C^2)} \geq 0 \quad (22)$$

$$\epsilon - v = \frac{1}{3} a (c - C) (b + C) (2b - c - C) \frac{3b^2 - 2cC - C^2}{b(-cC + b^2)(-5cC + 9b^2 - 2c^2 - 2C^2)} \quad (23)$$

Inequality (11) implies that the sign of  $\epsilon - v$  is the same as the sign of  $c - C$ .

### A.2 Monopoly markup

Equation 31 shows that when the monopoly sells to a single firm, it sets  $m < 0$  (a unit subsidy) if  $C < c$  and  $m > 0$  (a positive markup) if  $C > c$ . Equation 32 shows that the markup is always positive when the monopoly sells to two firms. In view of these two results,  $m^*(1) - m^*(2) < 0$  for  $C < c$ . To show that the markup with a single firm is larger than the markup with two firms when  $C$  is close to its upper bound,  $b$ , we need to compare the markup for large  $C$ . We have

$$\begin{aligned}
 m^*(1) - m^*(2) & = \frac{1}{6} a (b + C) (C - c) \frac{2b - C - c}{b(b^2 - Cc)} - \frac{1}{4} \frac{a(b - C)}{b} \\
 & = \frac{1}{12} \frac{a}{b(Cc - b^2)} [2C^3 - 2C^2b + 3C^2c - 7Cb^2 + Cbc - 2C^2c + 3b^3 + 4b^2c - 2bc^2].
 \end{aligned}$$



Evaluating the term in square brackets at  $C = b$  (the supremum of  $C$ ) we have

$$(2C^3 - 2C^2b + 3C^2c - 7Cb^2 + Cbc - 2Cc^2 + 3b^3 + 4b^2c - 2bc^2) = -4b(b - c)^2 < 0.$$

Because  $Cc - b^2 < 0$ , we conclude that for  $C$  close to (but smaller than)  $b$ ,  $m^*(1) - m^*(2) > 0$ .

### A.3 Effect of alternative assumption

We solve the optimization problem in Equation 21 to obtain the equilibrium value of  $m$  and substitute this into the equilibrium sales rules, and then substitute these into the inverse demand functions to obtain the equilibrium prices when the monopoly sells to a single agent. As noted above, our alternative assumption about the equilibrium bargain does not alter the equilibrium value of  $m$ , or the resulting prices, when the monopoly sells to two vendors. The difference in the generic price, in moving from one vendor to two vendors is

$$\frac{3}{2}a(c - C)^2(b + C) \frac{2b - c - C}{(9b^2 - 5cC - 2c^2 - 2C^2)(9b^2 - 7cC - c^2 - C^2)} > 0,$$

and the difference in the price of the new product is

$$\frac{3}{2}a(c - C)(b + C)(3b^2 - 2cC - C^2) \frac{2b - c - C}{b(9b^2 - 5cC - 2c^2 - 2C^2)(9b^2 - 7cC - c^2 - C^2)} > 0,$$

where the inequalities follow from Equation 11.

### A.4 Irrationality of alternative assumption

The discussion in the text provides the outline of the argument. We do not repeat the steps here, but merely show that the profit of the Stackelberg follower that sells a single product when the leader sells two products, is lower than the Cournot profit level when both firms sell a single product. To find the Stackelberg follower profit level, we solve the problem in Equation 21 to find  $m$ , substitute this value into the equilibrium rules for output, and then substitute these output levels into the profit function of the non-vendor, Stackelberg follower. The resulting profit level is

$$\frac{9}{4}a^2 \frac{(2bb - cb + Cb - cC - C^2)^2}{b(-c^2 - 7Cc - C^2 + 9bb)^2}.$$

The profit in the single-product Cournot equilibrium is  $\frac{a^2}{9b}$ . Thus, the decrease in profit in moving from the Cournot level to the Stackelberg follower level is

$$\frac{1}{36}a^2(c - C)(9b - 2c - 7C) \frac{-9bc + 9bC - 23cC + 36b^2 - 2c^2 - 11C^2}{b(9b^2 - 7cC - c^2 - C^2)^2}.$$

Given Inequality 11 and  $c > C$ , the sign of this expression is the same as the sign of

$$-9bc + 9bC - 23cC + 36b^2 - 2c^2 - 11C^2 = (9b - 23c)C - 11C^2 + (36b^2 - 9bc - 2c^2) \equiv H(C).$$

The quadratic on the right side of this equation, denoted  $H(C)$ , is concave in  $C$ , and  $H(0) > 0$  for all  $b, c$  that satisfy Inequality 11. Therefore  $H(C) > 0$  for  $0 \leq C \leq C^+$  where

$$C^+ = \frac{9}{22}b - \frac{23}{22}c + \frac{3}{22}\sqrt{185b^2 - 90bc + 49c^2}$$

is the positive root of  $H(C)$ . To complete the proof, we need only show that  $C^+ > b$ , so that  $H(C) > 0$  over the range that satisfies Inequality 11. We have  $C^+ > b$  if and only if

$$\frac{3}{22}\sqrt{185b^2 - 90bc + 49c^2} > \left(b - \frac{9}{22}b + \frac{23}{22}c\right),$$

which, with some manipulation, is equivalent to

$$\frac{88}{9}(b - c)(17b + c) > 0.$$

This inequality always holds because of Inequality 11.

## B Referees' appendix on consumer welfare

**Proposition 1** *If  $c > C$  then a representative consumer has higher utility when the monopoly sells to a single downstream firm. If  $c < C$  and  $C - c$  is “sufficiently small” (in a sense made precise in the proof) then consumers have higher welfare when the monopoly sells to two downstream firms. The consumer is indifferent between the two alternatives if  $c = C$ .*

The proof of this Proposition relies on two lemmas. The first lemma states that there is a linear relationship between the two aggregate quantities that holds regardless of whether the monopoly uses one or two vendors:

**Lemma 1** *For all values of  $m$ , aggregate sales in the generic market and the new market satisfy the relation*

$$\mathbf{q}_g = \frac{2a}{3b} - \frac{2c + C}{3b} \mathbf{q}_n. \quad (24)$$

**Proof.** (Lemma 1) When the upstream monopoly sells to both firms, we obtain the equilibrium conditions for aggregate sales in the two markets in a symmetric equilibrium, as functions of  $m$  using the general linear model from Appendix 1. We invert the formula for sales of the new product ( $j = n$ ) to obtain an expression for  $m$  as a function of aggregate sales,  $\mathbf{q}_n$ :

$$m = -\frac{1}{6} \frac{9b^2 - 2c^2 - 2C^2 - 5Cc}{b} \mathbf{q}_n - \frac{1}{3} \frac{2Ca + ca - 3ab}{b}. \quad (25)$$

We then substitute this equation into the equilibrium condition for aggregate generic sales,  $\mathbf{q}_g$ , as a function of  $m$  to obtain aggregate generic sales as a linear function of aggregate sales of the new product. The formula for this line is

$$\mathbf{q}_g = \frac{2a}{3b} - \frac{2c + C}{3b} \mathbf{q}_n. \quad (26)$$

This line summarizes the constraints implied by the equilibrium condition to the duopolists' quantity setting game. Given this constraint, a choice of, for example,  $\mathbf{q}_n$  determines the value of  $\mathbf{q}_g$  and also  $m$ . The values of these variables determine the monopoly's profits.

By its choice of  $m$ , the upstream monopoly selects a point on this line. The monopoly solves a maximization problem subject to two constraints, Equations 25 and 24.

When the monopoly sells to a single firm, its maximization is subject to the three equilibrium conditions: the two first-order conditions for generic sales and Firm 1's first-order condition for new product sales, which can be written as functions of  $m$ . We invert the equation for Firm 1's new-product sales to write  $m$  as a function of  $q_{n1} = \mathbf{q}_n$ . The result is

$$m = -\frac{1}{3} \frac{6b^2 - 4Cc - C^2 - c^2}{b} \mathbf{q}_n - \frac{1}{3} \frac{2Ca + ca - 3ab}{b}. \quad (27)$$

We use this equation to eliminate  $m$  from the remaining two equations (generic sales of the two firms) to obtain expressions for  $q_{g1}$  and  $q_{g2}$  as functions of  $\mathbf{q}_n$ . By adding the resulting two equations, we obtain the expression for aggregate generic sales as a function of aggregate new-product sales in Equation 24. ■

The next lemma notes several properties of the indirect utility function that stem from the linear relationship described by Lemma 1. Let  $V(p_g, p_n, y)$  be the indirect utility function for a representative agent,  $y$  be income, and  $\lambda$  (a function of prices and income) be the marginal utility of income:

**Lemma 2** (i) *Holding  $y$  fixed, as aggregate sales of  $\mathbf{q}_n$  increases and  $\mathbf{q}_g$  adjusts as specified by Equation 24, the change in utility is*

$$\frac{1}{\lambda} \frac{dV}{d\mathbf{q}_n} \Big|_{\text{Equation 24}} = \frac{1}{9} \left( \frac{9b^2 - 5Cc - 2C^2 - 2c^2}{b} \mathbf{q}_n + 2a \frac{c - C}{b} \right). \quad (28)$$

(ii) *For  $c \geq C$ ,  $dV/d\mathbf{q}_n > 0$  at every point on the line given by Equation 24, so utility reaches its maximum at the intercept of Equation 24*

$$(\mathbf{q}_g, \mathbf{q}_n) = \left( 0, \frac{2a}{2c + C} \right). \quad (29)$$

(iii) *For  $c < C$ ,  $dV/d\mathbf{q}_n < 0$  for small  $\mathbf{q}_n$ , and  $V$  reaches a minimum at an interior point on the line where*

$$\hat{\mathbf{q}}_n \equiv \frac{2a(C - c)}{9b^2 - 5Cc - 2C^2 - 2c^2}. \quad (30)$$

*The maximum of  $V$  might be at either intercept of Equation 24.*

**Proof.** (Lemma 2) (i) Totally differentiating the indirect utility function, holding  $y$  constant, dividing the result by  $\lambda$  and using Roy's identity implies

$$\begin{aligned} \frac{dV}{\lambda} &= \frac{1}{\lambda} \frac{\partial V}{\partial p_n} dp_n + \frac{1}{\lambda} \frac{\partial V}{\partial p_g} dp_g \\ &= -[\mathbf{q}_n dp_n + \mathbf{q}_g dp_g] \\ &= \mathbf{q}_n (bd\mathbf{q}_n + Cd\mathbf{q}_g) + \mathbf{q}_g (bd\mathbf{q}_g + cd\mathbf{q}_n) \end{aligned}$$

where the last line uses the total derivatives of the inverse demand equations (9). Divide both sides of the final equation by  $d\mathbf{q}_n$  to obtain

$$\frac{1}{\lambda} \frac{dV}{d\mathbf{q}_n} = \mathbf{q}_n \left( b + C \frac{d\mathbf{q}_g}{d\mathbf{q}_n} \right) + \mathbf{q}_g \left( b \frac{d\mathbf{q}_g}{d\mathbf{q}_n} + c \right).$$

Simplify this expression by using Equation 24 to eliminate  $\mathbf{q}_g$ , and noting that along the line in Equation 24,  $\frac{d\mathbf{q}_g}{d\mathbf{q}_n} = -\frac{2c+C}{3b}$ .

(ii) Because  $\lambda > 0$ , the sign of the right side of Equation 28 is the sign of the change in indirect utility due to an increase in  $\mathbf{q}_n$ , evaluated on the line given by Equation 24. By Inequality 11, the coefficient of  $\mathbf{q}_n$  on the right side of equation (28) is positive, so for  $c \geq C$ ,  $V$  is maximized at the corner given by Equation 29.

(iii) For  $c < C$ ,  $V$  is decreasing on this line in the neighborhood of the corner  $(\mathbf{q}_g, \mathbf{q}_n) = (\frac{2a}{3b}, 0)$ . Setting  $dV = 0$  implies Equation 30. Finally, we need to show that this value of  $\mathbf{q}_n$  is less than the value at the intercept,  $\frac{2a}{2c+C}$ . Subtracting these two values we have

$$\frac{2a(C-c)}{9b^2 - 5Cc - 2C^2 - 2c^2} - \frac{2a}{2c+C} = -6a \frac{3b^2 - 2Cc - C^2}{(9b^2 - 5Cc - 2C^2 - 2c^2)(2c+C)} < 0$$

where the inequality follows from Inequality 11. ■

**Proof.** (Proposition 1) The equilibrium level of new product sales when the upstream monopoly sells to a single firm, conditional on  $m$ , is

$$\frac{3ab - ca - 3mb - 2Ca}{6b^2 - 4Cc - C^2 - c^2},$$

and the monopoly's optimal level of  $m$  is

$$m^*(1) = \frac{1}{6}a(b+C)(C-c) \frac{2b-C-c}{b(b-Cc)}. \quad (31)$$

Conditional on the optimal level of  $m$ , the equilibrium level of new product sales with one vendor is

$$\mathbf{q}_n(1) \equiv \frac{1}{2} \frac{ab - Ca}{b^2 - cC}.$$

The equilibrium level of new product sales when the monopoly sells to two firms, conditional on  $m$ , is

$$-2 \left( \frac{2Ca + ca - ch - 3ab - 2Ch + 3mb}{9bb - 2c^2 - 2C^2 - 5Cc} \right)$$

and the optimal level of  $m$  is

$$m^*(2) = \frac{1}{4} \frac{a(b-C)}{b}. \quad (32)$$

Conditional on the optimal level of  $m$ , the equilibrium level of new product sales with two vendors is

$$\mathbf{q}_n(2) \equiv \frac{1}{2} a \frac{9b - 5C - 4c}{9b^2 - 2c^2 - 2C^2 - 5Cc}.$$

The difference between new product sales in the two cases is

$$\mathbf{q}_n(1) - \mathbf{q}_n(2) = a(b+C)(c-C) \frac{2b-c-C}{(b^2-Cc)(9b^2-2c^2-2C^2-5Cc)}.$$

Inequality (11) implies that the sign of  $\mathbf{q}_n(1) - \mathbf{q}_n(2)$  is the same as the sign of  $c - C$ .

If  $c > C$  so that  $\mathbf{q}_n(1) > \mathbf{q}_n(2)$ , consumer welfare is higher when the monopoly sells to a single firm, because utility is increasing in  $\mathbf{q}_n$  by Lemma 2 part (ii).

If  $C > c$  so that  $\mathbf{q}_n(1) < \mathbf{q}_n(2)$ , by Lemma 2 part (iii),  $V$  is increasing in  $\mathbf{q}_n$  for  $\mathbf{q}_n > \hat{\mathbf{q}}_n$ . Therefore, for  $C > c$ , a sufficient condition for consumers to be better off with two downstream firms selling the new product is  $\mathbf{q}_n(1) - \hat{\mathbf{q}}_n > 0$ . Using the definition of  $\hat{\mathbf{q}}_n$  in Equation (30) we have

$$\mathbf{q}_n(1) - \hat{\mathbf{q}}_n = \frac{1}{2}a \frac{\gamma}{(Cc - b^2)(-9b^2 + 2c^2 + 2C^2 + 5Cc)} \quad (33)$$

where

$$\gamma \equiv 9b^3 + (-13C + 4c)b^2 + (-2c^2 - 2C^2 - 5Cc)b - 2c^2C + 2C^3 + 9C^2c. \quad (34)$$

Because the denominator in the last line of Equation (33) is positive, a necessary and sufficient condition for  $\mathbf{q}_n(1) - \hat{\mathbf{q}}_n > 0$  is  $\gamma > 0$ . Define  $\varepsilon \equiv C - c > 0$  and write  $\gamma$  in terms of  $\varepsilon$ :

$$\gamma = 2\varepsilon^3 + (-2b + 15c)\varepsilon^2 + (-9bc - 13b^2 + 22c^2)\varepsilon + 9(b + c)(-c + b)^2.$$

This expression shows that for small  $\varepsilon$ ,  $\gamma > 0$ . A sufficient condition for  $\gamma > 0$  is that  $\varepsilon$  is smaller than the smallest positive root of  $\gamma = 0$ .

If  $c = C$  then  $\mathbf{q}_n(1) = \mathbf{q}_n(2)$ , so sales of both the new and the generic product are the same regardless of whether the monopoly sells to one firm or two firms. Consequently, consumer welfare is also the same in the two cases. ■

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